LOCAL STABILITY OF ROTATING TOKAMAK PLASMAS

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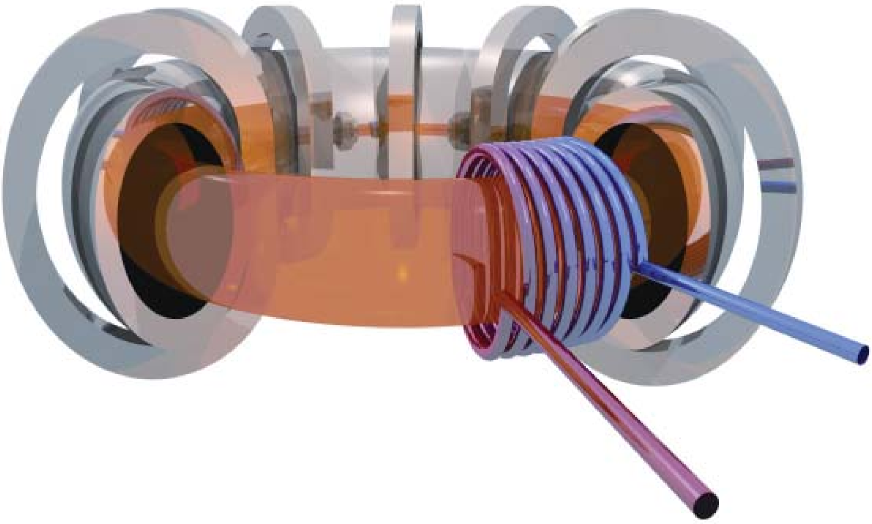
The most widely investigated design for a controlled nuclear fusion reactor for the production of energy, is that of the tokamak. This toroidal device (see figure) magnetically confines a hot (10^8 K) deuterium-tritium plasma. Present tokamaks often show toroidal plasma rotation approaching the sound speed. Such rotation can significantly influence the stability of the hot plasma, which can be described quite well with the theory of magnetohydrodynamics (MHD).

Using various simplifications, the MHD equations can be reduced to an inhomogeneous Sturm-Liouville type of differential equation [1]. We approached the question of marginal stability by investigating under which conditions solutions to this equation exist that neither oscillate nor grow. We linearized the angular rotation frequency Ω around a radial position *r* where the stabilizing influence of the magnetic field is smallest. Thereby we restricted the analysis to instabilities of limited spatial extent, excluding various spatially more extended instabilities. For subsonic flow, with ω_A the Alfvén frequency, the resulting stability criterion may be written as [2]

$$\underbrace{\frac{r}{4}\left(\frac{q'}{q}\right)^2 > -\beta'\left(1-q^2\right)}_{\text{Static plasmas [3, 4, 5]}} + \frac{q^2}{\omega_A^2}\left(\underbrace{\frac{1+2q^2}{4q^2} r\Omega'^2}_{\text{Kelvin-Helmholtz}} - \underbrace{\frac{\rho\Omega^{2\prime} + \Omega^2\rho'}{\rho}}_{\text{MRI + convective}} - \underbrace{\frac{\omega_{\mathrm{BV}}^2}{r}}_{\text{Brunt-Väisälä}} + \underbrace{2\Omega^{2\prime}}_{\text{flutter}} \right)$$



The first part of this stability criterion concerns static plasmas and dates back to the 1960s [3, 4, 5]. It states that the variation in the pitch q of the magnetic field and the magnetic field curvature $-\beta' q^2$ should be large enough to overcome the destabilizing pressure gradient $-\beta'$. The positive first term in between brackets represents the destabilizing Kelvin-Helmholtz effect of flow shear Ω'. The second term is typically destabilizing, because the kinetic energy density generally decreases radially in a tokamak plasma. This term can be decomposed into two terms that represent the magnetorotational instability (MRI) and the incompressible convective instability, respectively. The MRI plays a role in the turbulent transport of accretion disks.

The convective instability is well known from the atmosphere, which becomes unstable when the specific entropy decreases with height or otherwise facilitates stable oscillations with a Brunt-Väisälä (BV) frequency ω_{BV}. Here, the centrifugal force plays the role of gravity. The large magnetic pressure renders plasma motion perpendicular to the magnetic surfaces approximately incompressible, while motion within the magnetic surfaces can compress freely. This is why both the compressible (ω_{BV}^2) and incompressible ($r\Omega^2 \rho'/\rho$) BV-frequencies appear. The final term in the stability criterion is typically stabilizing and is associated with the fluttering motion of the plasma flow past the instability.

Although limited in its generality, the derived stability criterion provides great insight into the various rotational effects and provides quantitative information about their magnitude. The net influence of rotation can be stabilizing or destabilizing, depending crucially on the particular profiles of the angular frequency Ω and density ρ.

REFERENCES

1. C. Wahlberg. Plasma Phys. Contr. Fusion, vol. 51 (2009), 085006
2. J. W. Haverkort and H. J. de Blank. Plasma Phys. Contr. Fusion, vol. 53 (2011), 045008.
3. C. Mercier. Nucl. Fusion, vol. 1 (1960), pp. 47–53
4. A. A. Ware and F. A. Haas. Phys. Fluids, vol. 9 (1966), pp. 956–964
5. V. D. Shafranov and E. I. Yurchenko. Sov. Phys. JETP, vol. 26 (1968), pp. 682–686.